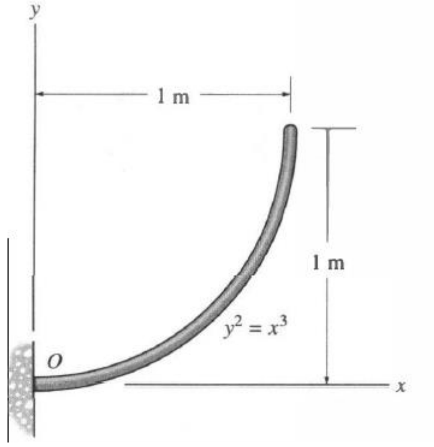
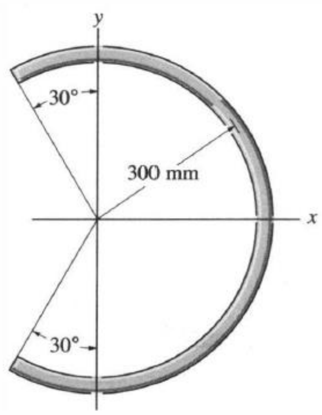
FACULTAD DE INEGENIERÍA MECÁNICA
ESTÁTICA. Taller Centroides

1. Determine la distancia $\bar{x}$ al centro de masa de la barra homogénea doblada en la forma que se muestra. Si la barra tiene una masa por longitud unitaria de 0.5 kg/m. Determine las reacciones en el soporte empotrado O. Rta: $\bar{x}$ = 0.546 m, Ox = 0, Oy = 7.06 N, Mo = 3.85 N · m.
2. Localice el centro de masa de la barra homogénea doblada en forma de un arco circular. Rta: $\bar{x}$ = 124 mm, $\bar{y}$ = 0.

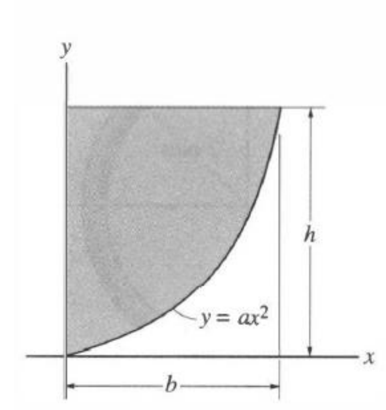


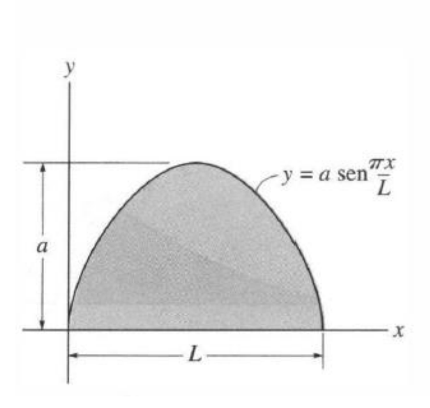
Ejercicio 1

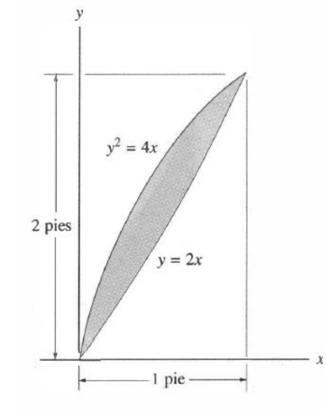


Ejercicio2

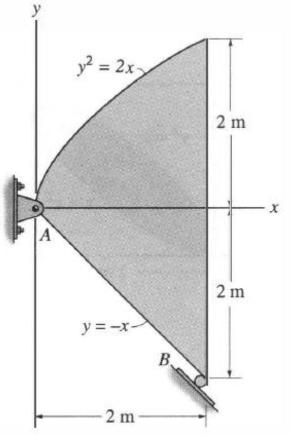
3. Localice los centroides de las áreas mostradas. Rta: $\bar{x}$ = 3b/8, $\bar{y}$ =3h/5 ; $\bar{x}$ = L/2, $\bar{y}$ =aπ/8; $\bar{y}$ = 1 pie.



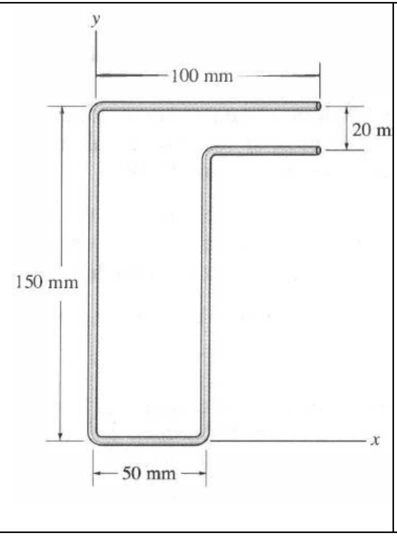


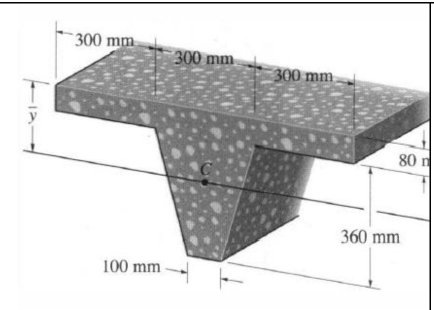


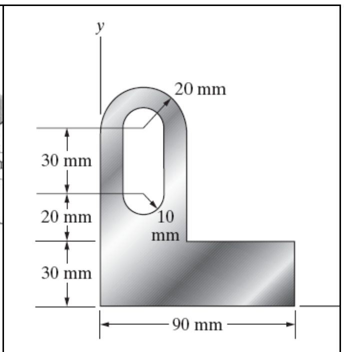
4. La placa de acero tiene un espesor de 0.3 m y densidad de 7850 kg/m^3. Determine la ubicación de su centro de masa. Calcule también las reacciones en el pasador y en el soporte de rodillo. Rta: $\bar{x}$ = 1.26 m, $\bar{y}$ = 0.143 m, N_B = 47.9 kN, Ax = 33.9 kN, Ay = 73.9 kN



5. Determine los centroides de los siguientes elementos compuestos. Rta: $\bar{x}$ = 34.4 mm, $\bar{y}$ = 85.8 mm; $\bar{y}$ = 135 mm; $\bar{x}$=35.3 mm, $\bar{y}$=33.2 mm







Integrales

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$$

$$\int \frac{dx}{a+bx} = \frac{1}{b}\ln(a+bx) + C$$

$$\int \frac{dx}{a+bx^2} = \frac{1}{2\sqrt{-ba}}\ln\left[\frac{a+x\sqrt{-ab}}{a-x\sqrt{-ab}}\right] + C,\quad ab<0$$

$$\int \frac{x\,dx}{a+bx^2} = \frac{1}{2b}\ln(bx^2+a) + C$$

$$\int \frac{x^2\,dx}{a+bx^2} = \frac{x}{b} - \frac{a}{b\sqrt{ab}}\tan^{-1}\frac{x\sqrt{ab}}{a} + C,\ ab>0$$

$$\int \sqrt{a+bx}\,dx = \frac{2}{3b}\sqrt{(a+bx)^3} + C$$

$$\int x\sqrt{a+bx}\,dx = \frac{-2(2a-3bx)\sqrt{(a+bx)^3}}{15b^2} + C$$

$$\int x^2\sqrt{a+bx}\,dx = \frac{2(8a^2-12abx+15b^2x^2)\sqrt{(a+bx)^3}}{105b^3} + C$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\,\mathrm{sen}^{-1}\frac{x}{a}\right] + C,\quad a>0$$

$$\int x\sqrt{a^2-x^2}\,dx = -\frac{1}{3}\sqrt{(a^2-x^2)^3} + C$$

$$\int x^2\sqrt{a^2-x^2}\,dx = -\frac{x}{4}\sqrt{(a^2-x^2)^3} + \frac{a^2}{8}\left(x\sqrt{a^2-x^2} + a^2\,\mathrm{sen}^{-1}\frac{x}{a}\right) + C,\ a>0$$

$$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right] + C$$

$$\int x\sqrt{x^2\pm a^2}\,dx = \frac{1}{3}\sqrt{(x^2\pm a^2)^3} + C$$

$$\int x^2\sqrt{x^2\pm a^2}\,dx = \frac{x}{4}\sqrt{(x^2\pm a^2)^3} \mp \frac{a^2}{8}x\sqrt{x^2\pm a^2} - \frac{a^4}{8}\ln\left(x+\sqrt{x^2\pm a^2}\right) + C$$

$$\int \frac{dx}{\sqrt{a+bx}} = \frac{2\sqrt{a+bx}}{b} + C$$

$$\int \frac{x\,dx}{\sqrt{x^2\pm a^2}} = \sqrt{x^2\pm a^2} + C$$

$$\int \frac{dx}{\sqrt{a+bx+cx^2}} = \frac{1}{\sqrt{c}}\ln\left[\sqrt{a+bx+cx^2} + x\sqrt{c} + \frac{b}{2\sqrt{c}}\right] + C,\ c>0$$
$$= \frac{1}{\sqrt{-c}}\,\mathrm{sen}^{-1}\left(\frac{-2cx-b}{\sqrt{b^2-4ac}}\right) + C,\ c<0$$

$$\int \mathrm{sen}\,x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \mathrm{sen}\,x + C$$

$$\int x\cos(ax)\,dx = \frac{1}{a^2}\cos(ax) + \frac{x}{a}\,\mathrm{sen}(ax) + C$$

$$\int x^2\cos(ax)\,dx = \frac{2x}{a^2}\cos(ax) + \frac{a^2x^2-2}{a^3}\,\mathrm{sen}(ax) + C$$

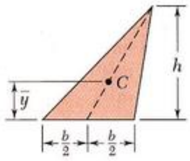
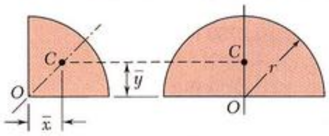
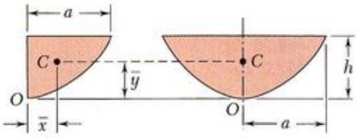
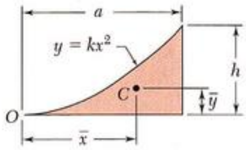
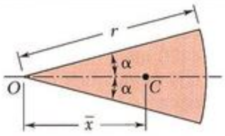
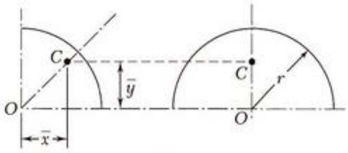
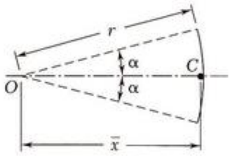
$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$$

$$\int x\,e^{ax}\,dx = \frac{e^{ax}}{a^2}(ax-1) + C$$

$$\int \mathrm{senh}\,x\,dx = \cosh x + C$$

$$\int \cosh x\,dx = \mathrm{senh}\,x + C$$

Centroides de áreas y líneas comunes

Forma		$\bar{x}$	$\bar{y}$	Área
Área triangular		$(1/3\ (a + b))$	$\frac{h}{3}$	$\frac{bh}{2}$
Un cuarto de área circular		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Área semicircular		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Área semiparabólica		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Área parabólica		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Tímpano parabólico		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
Sector circular		$\frac{2r \operatorname{sen} \alpha}{3\alpha}$	0	αr^2
Un cuarto de arco circular		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Arco semicircular		0	$\frac{2r}{\pi}$	πr
Arco de un círculo		$\frac{r \operatorname{sen} \alpha}{\alpha}$	0	$2\alpha r$